

Title	Estimation of fracture flow considering the inhomogeneous structure of single rock fractures
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Title: Estimation of fracture flow considering the inhomogeneous structure of single rock fractures

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Research Highlights

- In the available range of the cubic law, the fracture flow can be measured accurately.
- In a fracture flow simulation, the 2D model is more accurate than LCL.
- A high Re can be observed in some parts of the fracture flow in the available range of the cubic law.
- The inertia terms affect the fracture flow as an increment in Re .

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Abstract

Considering the safe, long-term isolation of energy byproducts, such as radioactive waste, one of the important parameters is the velocity of the groundwater flow through the void of rock masses and/or fractures. Although it is generally known that a natural rock fracture indicates a complex aperture distribution, the fracture is often ideally represented by a parallel plate model. The cubic law is applied to evaluate the hydraulic properties of fractured rock. From several previous research works, it is understood that the cubic law can be applied when the Reynolds number is less than 1.0 and that the inertia term can basically be ignored in such slow fracture flows. In this research work, two-dimensional seepage flow analyses, using the authors' proposed *2D* model, in which the inertia term, the pressure term and the diffusion term are incorporated, are carried out for single fracture permeability tests under conditions which allow for the application of the cubic law. In comparing the results of the experiments with the results of the numerical simulation, the results of the simulation employing the *2D* model show a good agreement with the experimental results; the *2D* model can simulate the water flow in an inhomogeneous fracture more accurately than the simulation based on the local cubic law. From these simulation results, the fracture flow in an inhomogeneous structure is discussed, along with the local Reynolds number, and the resistance through the fracture geometry is considered. Consequently, under the condition of a mean Reynolds number of less than 1.0, the inertia terms do not affect the fracture flow, but the hydraulic resistance does affect the fracture flow.

1. INTRODUCTION

The estimation of fracture flows through rock masses is important for discussions in many areas of geo-engineering, such as oil recovery, geothermal energy extraction, groundwater flow, and the underground disposal of anthropogenic CO₂ and radioactive waste. These flows dominantly occur in the interconnected systems of fractures. However, there are two reasons why the theory for the flows through rough surface rock fractures has not been fully developed, namely, the difficulty in defining the parameter values of the fracture distributions and the difficulty in understanding the fluid flow mechanics through individual fractures.

Generally, flows in fractures have been visualized conceptually as flows between parallel plates separated by an aperture (Snow, 1965; Iwai, 1976). In the parallel plate model, an individual fracture is represented by two infinitely smooth parallel plates, and the flow is assumed to be laminar with a parabolic velocity distribution across the aperture. Snow (1965) showed that the flow through an ideal parallel plate fracture was proportional to the cubes of the aperture, the well-known “cubic law”. Several studies have been conducted on the flows in rock fractures to investigate the validity of the cubic law. In considering the effect of aperture variation, the Reynolds equation is used to estimate the pressure distribution. The local cubic law (LCL), which assumes the cubic law can be applicable to each local void space to estimate the local transmissivity value, has been widely examined for calculating the discharge through inhomogeneous fractures (Walsh, 1981; Brown, 1987; Renshaw, 1995; Zimmerman and Bodvarsson, 1996; Brush and Thomson, 2003). The Reynolds equation was derived from the Navier-Stokes equation which assumes that most flows in real fractures are laminar and occur with a Reynolds number, namely, Re , of one or less (Zimmerman and Bodvarsson, 1996; Brush and Thomson, 2003). In other words, it is believed that the inertia term is negligible in laminar flows and under low Re conditions, and comparably, the viscous term significantly affects the flows in fractures. In addition, it is also assumed that the aperture changes gradually along the flow; and therefore, the effect of the characteristic length of

the aperture variation is also negligible. Furthermore, the pressure across the aperture is considered to be constant. In fact, the Reynolds equation only takes into account the effect of the shear strength of the viscous fluid at the fracture wall. The aperture is one of the significant parameters used to estimate the resistivity of fluid flows, transmissivity in this study. Therefore, the applicability of the Reynolds equation is limited to low Re conditions and smooth wall fractures along the flows (Al-Yaarubi, et al., 2005). Moreover, the variation in aperture occurs under high flow velocity, namely, high Re conditions, due to the rapid change in aperture in various parts of inhomogeneous fractures. Even if flow tests are conducted under conditions of $Re < 1$, neither the cubic law nor LCL can be satisfied.

Mgaya, et al. (2004; 2005) developed a 2D model, which was derived from both three-dimensional continuity and Navier-Stokes equations, and applied it to single fracture flow experiments (Kishida, et al., 2009). Through a comparison of the simulations and the experiments, the 2D model was found to show a good performance in terms of the fracture flow. However, such flow experiments have been conducted under a relatively high hydraulic head difference, Re of more than 1.0, and relatively wide aperture distributions due to the shear process.

In order to discuss the above-mentioned issues, accurate hydraulic tests through a fracture under the condition of a known aperture distribution and the geometry of the rock fracture walls are required. Measurements of the fracture aperture and discussions on the hydraulic conductivity in a single fracture, using the replica specimen, have been reported by several researchers. Hakami and Larsson (1996) conducted flow conductivity experiments and aperture measurements of single natural fractures in granite. Their technique for measuring the aperture was developed by utilizing the injection of a fluorescent epoxy into the fracture void space and taking measurements along sections across the fracture surface. Consequently, predictions of the flow rate using the measurement data were 2.4 times those of the experimental results. It is thought that the resolution along sections of the fracture surface was not sufficient. Yeo, et al. (1998) made aperture replicas of a natural sandstone fracture and directly measured it at 5.0 mm intervals. They also conducted flow

experiments and predictions using LCL and obtained aperture data. Their predictions of the flow rate were overestimated at around 1.7 to 2.0 times the experimental results. Konzuka and Kupper (2004) also carried out the same research works. Their predictions, through measured data and LCL, were overestimated at 1.7 times those of the experimental results.

On the other hand, an optical measurement method has been applied and accurate aperture data have been obtained (Nicholl, et al., 1999; Al-Yaarubi, et al., 2005). Nicholl, et al. (1999) measured an artificial regular aperture using the optical measurement system and showed that predictions through LCL were overestimated at 1.2 to 1.5 times those of the experiments. However, they did not apply it to the measurement of natural rock fractures. Al-Yaarubi, et al. (2005) measured the replica specimen of a natural sandstone fracture. They showed that the predictions of a fracture flow through LCL were estimated at around 1.2 to 1.4 times the flow experiments.

As mentioned above, accurate aperture data on inhomogeneous fractures and an elaborate numerical technique are necessary for discussing single rock fracture flows. In this research work, permeability tests have been carried out through a fracture under conditions for which the cubic law is applicable. And, the aperture and the fracture surface geometry have been measured under the same conditions as the permeability tests using the optical measurement system. Applying a precise simulation, including the effects of the inertia terms, the pressure term and the shear resistance from the fracture walls, the validity of the simulation has been discussed. Moreover, the fact that the factors significantly affect the fluid flow in a heterogeneous single fracture has been discussed based on the permeability tests and a 2D model simulation focusing on each flow factor, such as the inertia terms and the shear resistance from the fracture walls.

2. PERMEABILITY TESTS

2.1 *Specimen*

In this study, a transparent replica specimen for a single fracture is used and the aperture

measurement data, collected by Sato and Sawada (2010), are applied to calculate the flows in a single fracture. Sato and Sawada (2010) made an artificial tensile single fracture composed of two fracture walls in a granite block sample whose plane size is 100 mm by 100 mm. Then, a replica specimen was created by casting those two fracture walls of the granite fracture using a transparent epoxy resin (CRYSTAL RESIN2: NISSIN RESIN CO., Ltd.). **Photo 1** shows the transparent replica specimen, which includes a single fracture, by superposing two fracture wall copies of the granite fracture, consequently reproducing the spatially heterogeneous void space (aperture) distribution. The relative position between the two fracture walls for making the transparent replica was fixed by attaching injection and withdrawal ports for the permeability tests, as shown in **Photo 2**. The injection and withdrawal ports were attached at both ends along the flow direction for permeability and tracer tests, and the other two sides were fixed by plates to be no-flow boundaries. In this study, neither normal stress nor shear stress was applied to the fracture.

2.2 Measurement of fracture aperture

Upon measuring the spatial aperture distribution in the fracture of the transparent replica specimen, the optical measurement system, based on Beer-Lambert's law, is applied by filling a constant concentration of dyed fluid within the whole void space in the fracture. The heterogeneous spatial distribution of the strength of the lighting passing through the transparent replica specimen, perpendicular to the fracture plane, is measured with a high resolution (1.392 pixels by 1040 pixels) and 4069 gradation (12 bit) CCD sensor (DVC-1412AM), while the hydraulic aperture is measured by permeability tests. A band pass filter (central wavelength of 510 nm and half bandwidth of 10 nm) is also used to measure the monochromatic light intensity for applying Lambert-Beer's law. The measured strength of the lighting is affected by the dye concentration and the thickness of the

transmitted light which is the aperture at each point. Once the aperture spatial distribution was measured, the concentration of dye tracer at each spatial point could be estimated through the measured strength of the lighting. **Figure 1** shows the conceptual illustration of the optical measurement system of the spatial aperture distribution and the dye tracer concentration in the transparent replica specimen of a single fracture. The difference in refractive index between a fluid filling in the whole void space in the fracture and the transparent epoxy resin causes a light scatter at the interface between the fracture wall and the fluid, which significantly affects the accuracy of the aperture measurement at each point. In this research study, the refractive index matching method, which fills the fluid having a similar refractive index as the transparent epoxy resin into the fracture, was applied to measure the aperture distribution. A comparison of the refractive index between the transparent replica and the fluid (GM9002 base resin: Blenny Giken) used in the refractive matching method, under two different conditions for the wavelength of the light, listed in **Table 1**, shows closer values to each other under similar conditions of measuring light wave length as the absorption band of the dye fluid used in this study, 510 nm. **Photo 3** also shows a visual comparison between the two cases, a) filling water and b) filling the refractive index matching fluid in the whole void space in the transparent replica fracture. From **Photo 3**, the disturbance by the light scatter at the interface between the fracture wall and the fluid could be judged as less significant for the aperture spatial distribution measurement by applying the refractive index matching method. Based on this method, the spatial distribution of the fracture apertures of the transparent replica could be measured precisely, as shown in **Figure 2**. In this study, one pixel of CCD measurement represents 0.104 mm by 0.104 mm. The mean measured aperture is 0.33 mm.

For a comparison with the optical measurement system, the total void volume of the aperture is measured by the difference in weight between filling air and water in the whole void space in the fracture, and the mass balance aperture is measured through tracer tests. **Table 2** presents the mean fracture aperture obtained through several methods.

In this research study, flow simulations require not only an aperture distribution, but also the

geometry of the fracture surface. The topography of fracture surfaces of a pair of fracture surfaces, upper and lower, were measured by both a laser profilometer and the optical method. The former method has been widely used to measure fracture topography, but is disadvantageous in that the accuracy of the coordinate matching between the upper and the lower fracture walls may affect the estimated aperture distribution. In the later method, we applied the optical measurement system to measure the height of the transparent replica specimen, for which a part of the transparent replica was dyed by a constant concentration of dye and the attenuation of the light intensity through the dyed replica was measured. The method is advantageous in that both the aperture and the fracture topography can be measured under the same conditions and in the same coordinate system.

2.3 Hydraulic flow tests on a single fracture and the results

Hydraulic flow tests have been carried out under constant hydraulic gradient conditions, such as several hydraulic head differences of 0.25, 0.50, 0.75, 1.00 and 1.30 cm, respectively. The flow direction in the fracture is shown in **Figure 2** and the discharge water weight through the fracture has been measured by a precise electric scale.

Figure 3 shows the discharge-hydraulic head difference relation. This figure also shows the Re - hydraulic head difference relation. Here, Re is the mean value in a single fracture; it can be determined by the following equation:

$$Re = \frac{Q}{vw} \quad (1)$$

where Q is the discharge, v is the coefficient of kinematic viscosity and w is the width of the fracture specimen against the flow direction. In **Figure 3**, it can be confirmed that the discharge-hydraulic head difference relation is linear and it is thought that the experimental conditions are satisfied with Darcy's law. Moreover, the Re in each hydraulic head difference is observed to be less than 2.0 in these experimental results. It is thought that the experimental conditions, at least those with a hydraulic head difference of less than 0.75 cm, are satisfied with the cubic law conditions.

3. FLOW SIMULATIONS

3.1 Governing equation

Using the aperture distribution and the geometry of the fracture surface obtained through the optical measurement system, flow simulations through a single fracture have been carried out. In this research work, the 2D model developed by Mgya, et al. (2004, 2005) is applied. This model considers the effects of the inertia term, the pressure term and the shear stress of the fracture surface in the Navier-Stokes equations.

The governing equations are derived based on the 3D continuity and Navier-Stokes equations as follows:

$$\nabla \cdot (\rho \mathbf{u}) = 0 \quad (2)$$

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u u) = \rho \mathbf{F} - \nabla P + \frac{\partial}{\partial x} \tau_x + \frac{\partial}{\partial y} \tau_y + \frac{\partial}{\partial z} \tau_z \quad (3)$$

where ρ is the fluid density, \mathbf{u} is the velocity vector, \mathbf{F} is the body force vector, P is the hydrodynamic pressure, and τ_x , τ_y and τ_z are the viscous stress vectors acting at a point in the fluid on planes which are normal to the x -, y - and z - directions, respectively.

In the previous research work, a description of the Navier-Stokes equation being applied to a single fracture flow can be found. However, these past research works could not clarify how the inertia terms were integrated to include the advection terms, nor could they show how the diffusion terms were integrated to consider the kinematic boundary conditions. Zimmerman and Bodvarsson (1996) and Brush and Thomson (2003) showed that the cubic law could not be applied even to the slow flow condition. Based on these results, the inertia terms and the diffusion terms should be considered when discussing the fracture flow.

Kishida, et al. (2009) applied the depth average flow model to permeability tests on a single rock fracture. Unfortunately, the tests were conducted under the condition of a relatively high hydraulic head as when discussing the validity of the cubic law. Kishida et al. (2009) applied the assumptions

governing the derivation of the fracture flow as based on **Figure 4**. The assumptions governing the derivation are also described as follows: (i) the fluid is considered to be a Newtonian incompressible fluid and the flow is laminar, which means that non-turbulent conditions exist everywhere in the fracture; (ii) the fracture is free from any external forces such as compressive stress, which means that the fracture walls are fixed and no deformation occurs; (iii) neither slip nor flow occurs at the fracture walls; (iv) the difference between the true aperture, which is perpendicular to the fracture walls at each measured point, and the apparent aperture, which is defined as the void space along the z -direction, is negligible and the apparent aperture is used; (v) the velocity distribution perpendicular to the nominal fracture plane, in the z -direction in **Figure 4**, is parabolic; (vi) the pressure distribution along the aperture is hydrostatic and (vii) the inertia terms in the x - y plane, shown in **Figure 4**, are not negligible compared to the viscous terms. Appendices A and B present an integration of the 3D continuity and the Navier-Stokes equations from the lower wall to the upper wall, as shown in **Figure 4**. Consequently, the following governing equations for the 2D model are obtained:

[Continuity equation]

$$\frac{\partial UD}{\partial x} + \frac{\partial VD}{\partial y} = 0 \quad (4)$$

[Momentum equations]

$$\begin{aligned} \frac{\partial(UD)}{\partial t} + \beta \frac{\partial(U^2 D)}{\partial x} + \beta \frac{\partial(UVD)}{\partial y} = & -D \frac{\partial}{\partial x} \left(\frac{p_D}{\rho} \right) - gD \frac{\partial(z_b + D)}{\partial x} \\ & - \frac{\tau_{bx}}{\rho} \sqrt{1 + \left(\frac{\partial z_b}{\partial x} \right)^2 + \left(\frac{\partial z_b}{\partial y} \right)^2} - \frac{\tau_{sx}}{\rho} \sqrt{1 + \left(\frac{\partial z_s}{\partial x} \right)^2 + \left(\frac{\partial z_s}{\partial y} \right)^2} \\ & + \frac{\partial}{\partial x} \left(\nu D \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu D \frac{\partial U}{\partial y} \right) \end{aligned} \quad (5)$$

$$\begin{aligned}
& \frac{\partial(VD)}{\partial t} + \beta \frac{\partial(UVD)}{\partial x} + \beta \frac{\partial(V^2 D)}{\partial y} = -D \frac{\partial}{\partial y} \left(\frac{p_D}{\rho} \right) - gD \frac{\partial(z_b + D)}{\partial y} \\
& - \frac{\tau_{by}}{\rho} \sqrt{1 + \left(\frac{\partial z_b}{\partial x} \right)^2 + \left(\frac{\partial z_b}{\partial y} \right)^2} - \frac{\tau_{sy}}{\rho} \sqrt{1 + \left(\frac{\partial z_s}{\partial x} \right)^2 + \left(\frac{\partial z_s}{\partial y} \right)^2} \\
& + \frac{\partial}{\partial x} \left(\nu D \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu D \frac{\partial V}{\partial y} \right)
\end{aligned} \tag{6}$$

where U and V are the depth-averaged velocities in the x - and the y -directions, respectively. D is the aperture ($D = z_s - z_b$). z_s and z_b are the upper and the lower wall locations, respectively, from the datum plane. p_D is the pressure on the upper wall and g is the gravitational acceleration. τ_s and τ_b are the shear stress on the upper and the lower walls, respectively. ν is the kinematic viscosity and β is introduced here as a momentum correction factor for the laminar flow $\beta = 1.2$. The shear stress values on the fracture walls are calculated using the resistance law for laminar flow (i.e., $\tau_{bx}/\rho = \tau_{sx}/\rho = 6\nu U/D$; $\tau_{by}/\rho = \tau_{sy}/\rho = 6\nu V/D$).

Equations (4), (5), and (6) are discretized by the finite volume (FV) method with a staggered arrangement of the hydraulic variables. The velocity components are defined on the cell faces, while the scalar variables are defined at the cell centers. To calculate the pressure and the velocity field at the new time steps, the Highly Simplified Marker and Cell (HSMAC) method (Hirt and Cook, 1972) is applied.

3.2 Simulation results

Figure 5 shows the discharge–hydraulic head difference relation obtained from the experimental and the simulation results. Simulations through LCL have been conducted and the results are also plotted in **Figure 5**. These simulations can reproduce the results which have the same tendency as the experimental results, as shown in **Figure 5**. In addition, the 2D model is more advantageous than LCL. With increments in the hydraulic head difference, the difference between the 2D model and LCL increases. Mgaya, et al. (2004, 2005) showed that, with a Re of more than 1.0, the difference between the 2D model and LCL could be clearly confirmed. In this research work, with a hydraulic head difference of more than 0.75 cm, Re is more than 1.0. Therefore, a clear difference

between the 2D model and LCL can be confirmed in the high hydraulic head difference range; this is the same tendency as that seen in Mgaya, et al. (2004, 2005). In **Figure 5**, the ratios of the discharge obtained through the numerical simulation, Q_s , and through the experimental results, Q_e , are also plotted along the hydraulic head difference. From the ratios in **Figure 5**, LCLs are estimated to be more than 1.3 times those of the flow experiments, except in the case of a hydraulic head difference of 1.3 cm. These results are similar to those by Al-Yaarubi, et al. (2005). On the other hand, the results of the 2D model are estimated at less than 1.2 times the flow experiments, except in the case of a hydraulic head difference of 1.3 cm. The 2D model presents more accurate results than LCL. Regarding the terms which include the effect of the wall variation, LCL assumes the parallel plate model to be valid at local points; this means it does not consider the effect of the variation in fracture walls. Regardless of the fact that both models use the average aperture for the control volume, in the 2D model, terms $\partial z_s / \partial x$, $\partial z_s / \partial y$, $\partial z_b / \partial x$ and $\partial z_b / \partial y$ are included in the expressions for estimating the wall shear. Refer to **Equations (5) and (6)**, which indirectly include the inhomogeneous structure of fracture walls.

Figure 6 shows the distribution of flow vectors on the aperture contour map calculated by the 2D model. Although the flow on the fracture is relatively slow, the inhomogeneous flow distribution and the partial rapid flow can be observed. In **Figure 6**, an enlarged view is also shown. The enlarged view of the distribution of flow vectors shows the inhomogeneous flow distribution. Basically, the closed aperture disturbs the flow and the flows concentrate in the wide aperture area. And, from the wide aperture area, the fluid flows also concentrate to the run off. The difference in velocity between the aperture-closed area and the flow-concentrated area is confirmed as being too large. It is thought that rapid changes in flow velocity are not suitable for applying Darcy's law.

Figure 7 shows the flux and the aperture distribution along the six cross sections shown in **Figure 2** in the case of a hydraulic head difference of 0.5 cm. **Figure 7** also shows the mean flux in each cross section obtained through the experiments. Totally, the fluxes increase in the part of the wide aperture area and they are larger than those of the mean flux obtained through the experiments. In

each section, it is confirmed that the wider aperture area strongly affects the flow on the fracture. Along Line 1 in **Figure 7(a)**, the relatively wide aperture can be confirmed. However, the flow concentration cannot be observed and it is thought that the plane flow is shown such as in the experiments. In **Figures 7(b) and (d)**, the flow concentration into the wide aperture area can be observed. In **Figure 7(c)**, which is located at the downstream of **Figure 7(b)**, the flux becomes small at the downstream of the flux-concentrated area in **Figure 7(b)**. This phenomenon presents the flow separation from the wide aperture area. **Figures 7(e) and (f)** show the increment in flux at the wide aperture area and the flux can be observed to change depending on the changes in aperture. In each cross section, the fluxes are basically equal to or more than the mean fluxes through the experimental results.

4. DISCUSSION

4.1 *Re distribution in a single fracture*

Based on the velocity distribution in a single fracture, obtained through the 2D model, the *Re* distribution in a single fracture is calculated as follows:

$$Re = \frac{D'U'}{\nu}, \quad (7)$$

where U' is the mean velocity of the object relative to the fluid, D' is a characteristic linear dimension of the viscous forces and ν is the kinematic viscosity. Here, U' is used as the depth-averaged velocity along the x -direction (flow direction). As for D' , there are a couple of concepts for the flow in fractures. One is that D' is equal to 4 times the hydraulic radius, which is half of the fracture aperture (Lomize, 1951; Romm, 1966; Louis, 1969; Witherspoon, et al., 1980). The other is that D' is the aperture of the fractures (Zimmerman and Bodvarsson, 1996; Brush and Thomson, 2003; Kishida, et al., 2009). In this paper, the later concept is applied to a discussion on the experimental results, since it is considered that the aperture strongly affects the viscosity term of the fracture flow. Substituting the continuity equation, **Equation (5)** is expressed by **Equation (1)**.

Figure 8 shows the Re distribution in a single fracture. In the case of a small hydraulic head difference, it can be confirmed that a Re region of less than 1.0 occupies the majority of the fracture. However, a Re region of more than 1.0 can be found locally. On the other hand, in the case of a high hydraulic head difference, the Re region of less than 1.0 decreases and the Re region of more than 20 can be observed in the concentrated flow area. In this area, the cubic law cannot be applied to estimate the fracture flow. Consequently, the 2D model is more advantageous than LCL.

4.2 *Effect of inertia terms*

Mgaya, et al. (2004, 2005) and Kishida, et al. (2009) reported that the effect of the inertia terms appears in the region of the high Re range. In this research work, the experiments have been conducted under a relatively small Re range. Considering the effect of the inertia terms in this Re range, simulations in **Equations (5) and (6)**, where β is zero, have been conducted and the results are shown in **Figure 9**. **Figure 9(a)** shows the relationship between the hydraulic head difference and the discharge obtained through the experiments, the 2D model with and without the inertia terms and LCL. From **Figure 9(a)**, no clear difference can be observed between the simulations with the inertia terms and without the inertia terms. In **Figure 9(a)**, the simulations have been conducted in the case of relatively higher hydraulic head difference conditions, such as 2 cm and 4 cm. In these cases, the difference in discharge between the 2D model and LCL increases.

Figure 9(b) shows the relationship between the hydraulic conductivity simulated by the 2D model with and without the inertia terms and Re , which was obtained through the experiments. The hydraulic conductivity is expressed as the ratio of the discharge simulated by the 2D model to that of the LCL, such as Q_{2D}/Q_{LCL} , which can be investigated through the effect of the inertia terms. And, Q_{2D}/Q_{LCL} can also be indirectly related to a comparison between the 2D model and LCL. It is observed that as Re increases, the hydraulic conductivity for the 2D model with the inertia terms decreases. However, for the 2D model without the inertia terms, the hydraulic conductivity almost remains unchanged with increments in Re . This shows that the inertia terms become important as Re increases. Similar results were reported in previous research works (Brush and Thomson, 2003;

Mgaya, et al., 2006). On the other hand, the inertia terms are ignored in the 2D model; the ratio of Q_{2D}/Q_{LCL} is not equal to 1. This deviation is caused by the other terms, which are included in the 2D model presented in **Equations (5) and (6)**.

4.3 *Tracing of flow*

The flow paths from several points are traced using the hydraulic conductivity simulated by the 2D model. **Figure 10** shows the flow paths in cases of hydraulic head differences of 0.25 cm and 1.00 cm, respectively. In these figures, it cannot be observed that each flow path is tortuous and is inhomogeneous. **Figure 10** also shows the ratio between the length of the flow path and the straight line distance, L_d/L . From these ratios, the flow path is 3 to 9% longer than the straight line. In general, the hydraulic gradient is defined by the ratio of the hydraulic head difference and it is estimated using the specimen size along the flow direction. Based on these results, the hydro gradient is actually smaller than that obtained through the specimen size.

Travel times of the flow paths are not constant. The travel time located in the upper part of **Figure 10** is relatively faster than that located in the lower part of **Figure 10**. This is confirmed by the tracer experiments shown in **Figure 11** (Sawada and Sato, 2010). The results of **Figure 10** show a good agreement with those of **Figure 11**. And the heterogeneous distribution of the flow paths can be confirmed from both results.

4.4 *Effect of contact area*

From the Re distributions (**Figure 8**) and the flow tracing (**Figure 10**), it can be observed that the flow of the fracture is heterogeneous. Several researchers explained the heterogeneous distribution of flow by considering the regions where the fracture is open and by treating the contact regions with separate methods (Walsh, 1981; Piggott and Elsworth, 1992). Zimmerman and Bodvarsson (1996) reviewed the relation between the geometrical aperture and the hydraulic aperture for which the influence of obstructions was considered. Walsh (1981) explained that the hydraulic aperture can be expressed as

$$D_h^3 = [(1-c)/(1+c)] D_m^3, \quad (8)$$

where c is the areal fraction of the fracture plane that is occupied by the obstructions, and D_h and D_m are the hydraulic aperture and the nominal aperture, respectively. In this research, the representative aperture value measured by the optical method shown in **Table 2** can be represent D_m . Here, c , the obstruction area of the fracture flow, is estimated by the following two methods. In one method, c is estimated by the measured aperture. The threshold values of the measured aperture are assumed and the area where there is a small aperture of some threshold values is estimated as the obstruction area. In the other method, Re is applied to estimate parameter c . The area where Re presented small values is estimated as the obstruction area. **Figure 12** shows the relationship between parameter $(1-c)/(1+c)$ and the hydraulic head difference. In the case of c defined by Re , parameter $(1-c)/(1+c)$ rapidly increases along the increment of the hydraulic head difference. In the range of Re of more than 1.0, obtained from permeability tests, as shown in **Figure 3**, $(1-c)/(1+c)$ reached a constant value. In the case of c defined by the fracture aperture, on the other hand, $(1-c)/(1+c)$ is constant for each hydraulic head difference. In this case, $(1-c)/(1+c)$ does not depend on the flow. In comparing the two types of $(1-c)/(1+c)$, parameter $(1-c)/(1+c)$, estimated by Re , is in agreement with the one estimated by a threshold aperture of 0.0125 cm in the range of a hydraulic head difference of more than 0.75 cm. As mentioned in **Equation 7**, Re is defined by the flow velocity and the aperture. In the area which presented smaller Re , the external slow flow and/or a smaller aperture is occupied. In the range of Re of more than 1.0, obtained through the permeability tests, it is thought that the main factor of the flow obstructions depends on the aperture distribution. On the other hand, even if the flow has a Re of less than 1.0, the difference in local hydraulic head occurs due to the asperities of the joint surface roughness and the wall friction is affected by the fracture flow.

5. CONCLUSION

The numerical simulation of a flow through a single fracture has been conducted under relatively

slow flow conditions which satisfy the cubic law. And, the local flow of the fracture and the validity of the cubic law have been discussed. Consequently, it has been found that the 2D model is more advantageous for fracture flow simulations under a relatively small Re range than LCL. Under conditions of a small mean Re , a high Re of more than 1.0 can be locally found in the fracture. The cubic law is not satisfied for the local points of the fracture, and the flows in these areas strongly affect the flows through the fracture. In this Re range, the hydraulic conductivity is expressed as the ratio of the discharge simulated by the 2D model to that of the LCL, such as Q_{2D}/Q_{LCL} , which can be investigated by the effect of inertia terms. Consequently, when Re increases, it is confirmed that Q_{2D}/Q_{LCL} with inertia terms decreases and that without inertia terms is constant. It is also confirmed that the inertia terms affect the flows through the fracture. In a range in Re of less than 1.0, the influence of the inertia terms exists. However, it is relatively small. Moreover, it is thought that the variation in pressure and the resistance of the fracture walls may affect the flows through the fracture.

From the numerical simulations assuming the steady flow condition, the flow path is 3 to 9% longer than the straight line in this specimen. This increment in the flow path occurs due to the distribution of the contact and the non-contact areas. Consequently, the hydro gradient is actually smaller than that obtained through the specimen size. These findings comprise one of the reasons why the cubic law cannot be easily applied to evaluate the experimental results.

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Appendix A

The continuity equation for the 2D model is derived by firstly rewriting the following form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (\text{a1}),$$

where u , v and w are the velocities in the x -, y - and z - axes, respectively. Integrating Equation (a1) from the lower wall to the upper wall, as shown in Figure 4, and applying the Leibniz theorem, the following expressions are obtained:

$$\int_{z_b}^{z_s} \frac{\partial u}{\partial x} dz = \frac{\partial}{\partial x} \int_{z_b}^{z_s} u dz - u_s \frac{\partial z_s}{\partial x} + u_b \frac{\partial z_b}{\partial x} \quad (\text{a2})$$

$$\int_{z_b}^{z_s} \frac{\partial v}{\partial y} dz = \frac{\partial}{\partial y} \int_{z_b}^{z_s} v dz - v_s \frac{\partial z_s}{\partial y} + v_b \frac{\partial z_b}{\partial y} \quad (\text{a3})$$

$$\int_{z_b}^{z_s} \frac{\partial w}{\partial z} dz = w_s - w_b \quad (\text{a4}),$$

where subscripts s and b denote variables or parameters at the upper and the lower walls of the fracture, respectively. Using the kinetic boundary conditions at the lower and the upper fracture walls,

$$w_s = \frac{\partial z_s}{\partial t} + u_s \frac{\partial z_s}{\partial x} + v_s \frac{\partial z_s}{\partial y} \quad (\text{a5})$$

$$w_b = \frac{\partial z_b}{\partial t} + u_b \frac{\partial z_b}{\partial x} + v_b \frac{\partial z_b}{\partial y} \quad (\text{a6}),$$

and considering the fact that the boundary walls of the fractures are fixed such as $dz_s/dt = dz_b/dt = 0$,

it follows that

$$\int_{z_b}^{z_s} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dz = \frac{\partial}{\partial x} \int_{z_b}^{z_s} u dz + \frac{\partial}{\partial y} \int_{z_b}^{z_s} v dz \quad (\text{a7})$$

Then, Equation (4) can be obtained.

Appendix B

The momentum equation, Equation (3), in the x -direction can be written as follows:

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left(\frac{\tau_{xx}}{\rho} \right) + \frac{\partial}{\partial y} \left(\frac{\tau_{xy}}{\rho} \right) + \frac{\partial}{\partial z} \left(\frac{\tau_{xz}}{\rho} \right) \quad (\text{a8})$$

where ρ is the fluid density and p is the driving force, in this case, pressure. Note that body force \mathbf{F} in Equation (3) disappears since the pressure in Equation a8 is defined as a reduced pressure, namely, $p = P + \rho gz$.

For simplicity and clarity, the derivation of the momentum equation, Equation (a8) is divided into three parts, namely, the inertia term on the left-hand side of Equation (a8), the pressure term as the first term on the right-hand side of Equation a8, and the diffusion term on the right-hand side of Equation (a8) without the pressure term, respectively.

Integrating the inertia terms on the left-hand side of Equation a8 by applying the Leibniz theorem in Figure 4, it follows that

$$\begin{aligned} \int_{z_b}^{z_s} \frac{\partial u}{\partial t} dz &= \frac{\partial}{\partial t} \int_{z_b}^{z_s} u dz - u_s \frac{\partial z_s}{\partial t} + u_b \frac{\partial z_b}{\partial t} \\ \int_{z_b}^{z_s} \frac{\partial uu}{\partial x} dz &= \frac{\partial}{\partial x} \int_{z_b}^{z_s} u u dz - u_s u_s \frac{\partial z_s}{\partial x} + u_b u_b \frac{\partial z_b}{\partial x} \\ \int_{z_b}^{z_s} \frac{\partial uv}{\partial y} dz &= \frac{\partial}{\partial y} \int_{z_b}^{z_s} u v dz - u_s v_s \frac{\partial z_s}{\partial y} + u_b v_b \frac{\partial z_b}{\partial y} , \\ \int_{z_b}^{z_s} \frac{\partial uw}{\partial z} dz &= u_s w_s \frac{\partial z_s}{\partial y} - u_b w_b \frac{\partial z_b}{\partial y} \end{aligned} \quad (\text{a9})$$

Applying the kinetic boundary conditions, such as Equations a5 and a6, with the parabolic velocity assumption, the following relation is derived:

$$\int_{z_b}^{z_s} \left(\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} \right) dz = \frac{\partial}{\partial t} \int_{z_b}^{z_s} u dz + \frac{\partial}{\partial x} \int_{z_b}^{z_s} u u dz + \frac{\partial}{\partial y} \int_{z_b}^{z_s} u v dz \quad (\text{a10})$$

Then, the expression for the inertia terms can be written as follows:

$$\int_{z_b}^{z_s} \left(\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} \right) dz = \frac{\partial(UD)}{\partial t} + \beta \frac{\partial(DU^2)}{\partial x} + \beta \frac{\partial(DVU)}{\partial y} \quad (\text{a11})$$

where U and V are the depth-averaged velocities in the x - and y -directions, respectively. D is the

aperture ($D = z_s - z_b$).

The expression for pressure is derived based on the assumption that the pressure distribution perpendicular to the nominal fracture plane is hydrostatic.

$$-\int_{z_b}^{z_s} \frac{\partial}{\partial x} \left(\frac{p}{\rho} \right) dz = -\frac{\partial}{\partial x} \int_{z_b}^{z_s} \frac{p}{\rho} dz + \left(\frac{p}{\rho} \right)_{z_s} \frac{\partial z_s}{\partial x} - \left(\frac{p}{\rho} \right)_{z_b} \frac{\partial z_b}{\partial x} \quad (\text{a12})$$

Using the relation for the pressure at depth z from the datum,

$$p = p_D + \rho g(z_s - z) \quad (\text{a13})$$

where p_D is the pressure on the upper wall and g is the gravitational acceleration.

Then, the following expression can be derived after some manipulation:

$$-\int_{z_b}^{z_s} \frac{\partial}{\partial x} \left(\frac{p}{\rho} \right) dz = -\frac{D}{\rho} \frac{\partial p_D}{\partial x} - gD \frac{\partial (D + z_b)}{\partial x} \quad (\text{a13})$$

Integrating the diffusion term on the right-hand side of Equation a8, by applying the Leibniz theorem, it follows that

$$\begin{aligned} \int_{z_b}^{z_s} \frac{\partial}{\partial x} \left(\frac{\tau_{xx}}{\rho} \right) dz &= \frac{\partial}{\partial x} \int_{z_b}^{z_s} \frac{\tau_{xx}}{\rho} dz - \left(\frac{\tau_{xx}}{\rho} \right)_s \frac{\partial z_s}{\partial x} + \left(\frac{\tau_{xx}}{\rho} \right)_b \frac{\partial z_b}{\partial x} \\ \int_{z_b}^{z_s} \frac{\partial}{\partial y} \left(\frac{\tau_{xy}}{\rho} \right) dz &= \frac{\partial}{\partial y} \int_{z_b}^{z_s} \frac{\tau_{xy}}{\rho} dz - \left(\frac{\tau_{xy}}{\rho} \right)_s \frac{\partial z_s}{\partial y} + \left(\frac{\tau_{xy}}{\rho} \right)_b \frac{\partial z_b}{\partial y} \\ \int_{z_b}^{z_s} \frac{\partial}{\partial z} \left(\frac{\tau_{xz}}{\rho} \right) dz &= \left(\frac{\tau_{xz}}{\rho} \right)_s - \left(\frac{\tau_{xz}}{\rho} \right)_b \end{aligned} \quad (\text{a14})$$

Using the definition of shear stress,

$$\frac{\tau_{xx}}{\rho} = 2\nu \frac{\partial u}{\partial x}, \quad \frac{\tau_{xy}}{\rho} = \nu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (\text{a15})$$

and after some manipulation, the following expression is obtained:

$$\frac{\partial}{\partial x} \int_{z_b}^{z_s} \frac{\tau_{xx}}{\rho} dz + \frac{\partial}{\partial y} \int_{z_b}^{z_s} \frac{\tau_{xy}}{\rho} dz = \frac{\partial}{\partial x} \left(\nu D \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu D \frac{\partial U}{\partial x} \right) \quad (\text{a12})$$

Consequently, the momentum equation for the 2D model is summarized in Equations 5 and 6.